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Shock Waves

Shock wave interaction with cellular materials

Part I: analytical investigation and governing equations

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Abstract. The equations governing the head-on collision of a planar shock wave with a cellular material and a numerical scheme for solving the set of the governing equations were outlined. In addition, the condition for the transmitted compression waves to transform into a shock wave, inside the cellular material was introduced. It was proved analytically that a cellular material cannot be used as a means of reducing the pressure load acting on the end-wall of the shock tube. In subsequent papers, the interaction of planar shock waves with specific cellular materials, e.g., foams and honeycombs will be described in detail.

Key words: Analysis, Cellular material, Shock wave propagation, Two-phase flow

1. Introduction

Recent publications indicate that there is a growing interest in the interaction of shock waves with porous materials, e.g., Gelfand et al. (1983), Grozdeva et al. (1985), Gvozdeva et al. (1986), van der Griten et al. (1988), Korobeinikov and Urtiew (1989), Henderson et al. (1990), Skews (1991), Ben-Dor et al. (1992), Seitz and Skews (1992), and Skews et al. (1992). Unfortunately, however, by using the general term "porous materials" investigators have neglected the fact that the interaction of shock waves with porous materials depends strongly on the internal structure of the porous material which determines its physical properties. For example, two foams which are made of an identical material and have identical bulk densities would react differently to a head-on colliding shock wave if one had an open-cell structure and the other a closed-cell structure (these terms are explained subsequently).

As a consequence, the present paper deals with the interaction of planar shock waves with cellular materials. It is devoted to analytical investigation and formulation of the governing equations, and subsequent papers will describe numerical as well as experimental results of shock wave interactions with various cellular materials.

A cellular material as defined by Gibson and Ashby (1988) is a material "made up of a interconnected network of solid struts or plates which form the edges and faces of cells." In general cellular materials can be divided into two typical structures: honeycombs – polygonal cells which pack in two-dimensions to fill a plane area like the hexagonal cells of the beehive; foams – polyhedra cells which pack in three-dimensions to fill a space. The foams can be further subdivided into: open celled foams in which the cells connect through open faces; and closed celled foams in which each cell is sealed off from its neighbor cells.

The most important property of a cellular solid is its relative density, ρ_c/ρ_s, i.e., the density of the cellular material, ρ_c, divided by the density of the solid material from which the cell walls are made, ρ_s. Relative densities as low as 0.001 can be found in special ultra low density foams. Most commercially used foams, i.e., polymeric foams, have relative densities in the range $0.05 \leq \rho_c/\rho_s \leq 0.20$, cork is about 0.14 and most softwoods have relative densities in the range of $0.15 \leq \rho_c/\rho_s \leq 0.40$. As the relative density increases the walls of the cell thickens and the pore space shrinks. When the relative density increases beyond 0.3, there is a transition from a cellular structure to one of a solid containing isolated pores. The present paper deals with the interaction of shock waves with true cellular solids and hence is limited to relative densities less than 0.3, i.e., $\rho_c/\rho_s < 0.3$.

In the following, the equations governing the phenomenon under consideration, i.e., the head-on interaction of a planar shock wave with a cellular material are developed. The development is followed by a discussion on the dependence of the phenomenon on the stress-strain relation of the cellular material, which for example determines whether or not compression waves transform to a shock wave, as they propagate along the cellular material. Then an analytical proof that shock wave loadings cannot be reduced by means of cellular materials is presented. Finally, a brief description of the numerical method used in the course of the present study for solving the governing equations is

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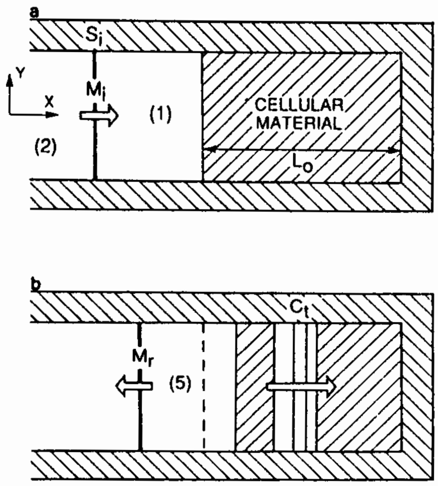


Fig. 1a,b. Schematic illustration of the problem under consideration: **a** prior to the head-on collision; **b** immediately after the head-on collision

given. Numerical and experimental results of the head-on interaction of shock waves with specific cellular materials, e.g., elastomeric foams, elastic-plastic foams, elastic-brittle foams, etc. will be given in subsequent papers.

2. Theoretical background

The problem under consideration is shown schematically in Fig. 1. In Fig. 1a an incident planar shock wave, S_i, having a constant Mach number M_i, is seen to propagate in a quiescent gas, state (1), inside a shock tube, towards a cellular material which is supported at its rear by the end-wall of the shock tube. The cellular material completely fills the shock tube cross section. At some time, t, the shock wave collides, heads on, with the front edge of the cellular material. As a result a shock wave, S_r, having a Mach number, M_r, is reflected into the gas and a compression wave, C_t, is transmitted into the cellular material. This is shown schematically in Fig. 1b.

Since the expansion of the cellular materials towards the y- and z-direction is limited by the side walls of the shock tube, it experiences a uni-axial strain compression, i.e., $\varepsilon_x \neq 0$, $\varepsilon_y = 0$, and $\varepsilon_z = 0$, where ε_i is the strain in the i-th direction. As a consequence, stresses are developed in both the x-, y-, and z-directions, i.e., $\sigma_x \neq 0$, $\sigma_y \neq 0$, and $\sigma_z \neq 0$.

The compression of the cellular material in the x-direction is caused by means of the transmitted compression wave. Depending upon the stress-strain relation of the specific cellular material, σ-ε, as well as its length, L, the transmitted compression wave might transform to a shock wave. This will be discussed, in more detail, subsequently.

3. The assumptions

In the following, the governing equations appropriate to the considered problem are developed using a Lagrangian approach. For details see Mazor et al. (1992). The development is based on the following assumptions:

(1) The problem is one-dimensional, and nonstationary.
(2) The gas is an ideal fluid, i.e., inviscid ($\mu = 0$) and thermally non-conductive ($k = 0$).
(3) The gas behaves as a perfect gas, i.e., its equation of state is $P = \rho RT$ and its internal energy, e, is given by $e = C_v T$, where P, ρ, and T are the gas pressure, density, and temperature, respectively, C_v is its specific heat capacity at a constant volume and R is its specific gas constant.
(4) All body forces are neglected.
(5) The cellular material is a uniform solid phase in which the mechanical properties are related to its structure and to the material properties from which the cell walls of the pores of the cellular material are made. Thus its mechanical properties are expressed as functions of its relative density – ρ_c/ρ_s.
(6) The cellular material is isotropic and compressible.
(7) The friction forces acting on the external surfaces of the cellular material are neglected.
(8) The stresses which develop in the cellular material are uniformly distributed inside any cross-sectional area perpendicular to the x-axis. Therefore, the cross-sectional area remains planar throughout the deformation stages of the cellular material.
(9) The gaseous phase does not penetrate the cellular material at the contact surface between them. Note that this assumption is completely satisfied in closed-cell foams.

4. The governing equations

Based on the above listed assumptions, the governing equations for the gaseous phase are:
conservation of mass;

$$\frac{\partial}{\partial t}\left(\rho_g(h_g,t)\frac{\partial x(h_g,t)}{\partial h_g}\right) = 0, \tag{1}$$

definition of the gas velocity;

$$U_g(h_g,t) = \frac{\partial x(h_g,t)}{\partial t}, \tag{2}$$

conservation of linear momentum;

$$\frac{\partial U_g(h_g,t)}{\partial t} = -A\frac{\partial P(h_g,t)}{\partial h_g}, \tag{3}$$

conservation of energy;

$$\frac{\partial e(h_g,t)}{\partial t} = -P(h_g,t)\frac{\partial V_g(h_g,t)}{\partial t} \tag{4}$$

where e, the internal energy, is given by;

$$e(h_g,t) = C_v T(h_g,t)$$

and V_g, the specific volume, is given by $V_g = 1/\rho_g$.
The equation of state is;

$$P(h_g,t) = \frac{RT(h_g,t)}{V_g(h_g,t)}. \tag{5}$$

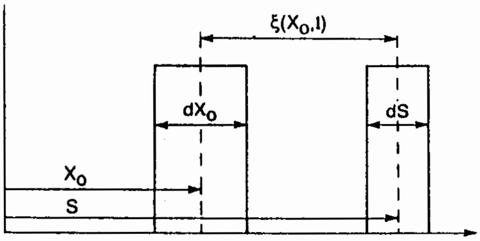
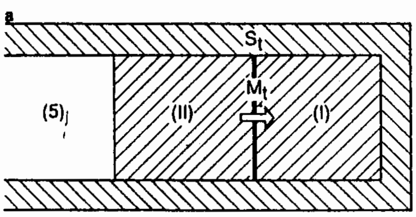


Fig. 2. Schematic illustration of a mass element before and after its deformation



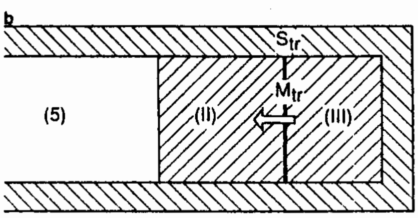


Fig. 3a,b. Schematic illustration of the head-on reflection of the shock wave, transmitted into the cellular material, at the end-wall of the shock tube: **a** prior to the reflection; **b** immediately after the reflection

In the above equations h_g is the Lagrangian variable for the gaseous phase. Its definition is,

$$h_g = \int_{x(0,t)}^{x(h_g,t)} \rho_g A \mathrm{d}x.$$

The governing equations for the cellular material are: conservation of mass,

$$\frac{\partial}{\partial t}\left(\rho_c(h_c,t)\frac{\partial S(h_c,t)}{\partial h_c}\right) = 0. \tag{6}$$

For defining a particle velocity inside the cellular material let us consider a mass element dm which at $t = 0$ was at $x = X_0$ and had a width $\mathrm{d}X_0$ (see Fig. 2). Due to the deformation of the cellular material, this mass element moved to a new position, S, and its width changed to $\mathrm{d}S$. It is apparent from Fig. 2 that $S = X_0 + \xi$ where ξ is the magnitude of the displacement along the S-axis. The cellular material velocity may therefore be defined as,

$$U_c(h_c,t) = \frac{\partial S(h_c,t)}{\partial t} \text{ or } U_c(h_c,t) = \frac{\partial \xi(h_c,t)}{\partial t}. \tag{7}$$

The extension ratio in the x-direction, λ_x, which expresses the ratio between the mass elements width after and prior to deformation is

$$\lambda_x(h_c,t) = \frac{\partial S(h_c,t)}{\partial X_0}.$$

With the addition of $\dfrac{\partial S(h_c,t)}{\partial X_0} = \dfrac{\partial S(h_c,t)}{\partial h_c}\dfrac{\partial h_c}{\partial X_0}$ and the definition of the Lagrangian variable for the cellular material, h_c, i.e.,

$$h_c = \int_{S(0,t)}^{s(h_c,t)} \rho_c A \mathrm{d}S$$

the above mentioned expression for λ_x could be rewritten as,

$$\lambda_x(h_c,t) = \rho_c^0 A \frac{\partial S(h_c,t)}{\partial h_c}$$

where the superscript 0 indicates pre-deformation values. For a material under compressive loads $0 < \lambda_x < 1$ and for a material under tensile loads $\lambda_x > 1$. When the material is load free (i.e., stress free conditions) $\lambda_x = 1$. The strain in the x-direction can be simply obtained from the definition $\varepsilon_x = 1 - \lambda_x$, i.e.,

$$\varepsilon_x(h_c,t) = 1 - \rho_c^0 A \frac{\partial S(h_c,t)}{\partial h_c}. \tag{8}$$

For a mass element $\mathrm{d}m$ where $\mathrm{d}m = \rho_c A \mathrm{d}S = \rho_c^0 A \mathrm{d}X_0$, Newton's second law implies

$$\frac{\partial}{\partial t}(\mathrm{d}m U_c) = -\frac{\partial F_x}{\partial X_0}\mathrm{d}X_0$$

where F_x is the normal force acting on a given cross section at time t. Substituting (7) with the definition of the mass element into the above expression results in,

$$\rho_c^0 A \frac{\partial^2 \xi(h_c,t)}{\partial t^2} = -\frac{\partial F_x(f_c,t)}{\partial X_0}.$$

The above expression represents the conservation of momentum of the cellular material.

By introducing the stress in the x-direction,

$$\sigma_x(h_c,t) = \frac{F_x(h_c,t)}{A}$$

the above expression becomes

$$\frac{\partial^2 \xi(h_c,t)}{\partial t^2} = -\frac{1}{\rho_c^0}\frac{\partial \sigma_x(h_c,t)}{\partial X_0}.$$

However, since

$$\frac{\partial \sigma_x(h_c,t)}{\partial X_0} = \frac{\partial \sigma_x(h_c,t)}{\partial h_c}\frac{\partial h_c}{\partial X_0} = \frac{\partial \sigma_x(h_c,t)}{\partial h_c}\rho_c^0 A$$

one obtains

$$\frac{\partial^2 \xi(h_c,t)}{\partial t^2} = -A\frac{\partial \sigma_x(h_c,t)}{\partial h_c}.$$

But as shown in Fig. 2, $S = X_0 + \xi$, hence the equation of motion of the cellular material is,

$$\frac{\partial^2 S(h_c,t)}{\partial t^2} = -A\frac{\partial \sigma_x(h_c,t)}{\partial h_c} \tag{9a}$$

or alternatively, together with (7),

$$\frac{\partial U_c(h_c,t)}{\partial t} = -A\frac{\partial \sigma_x(h_c,t)}{\partial h_c}. \tag{9b}$$

Equations (1) to (9) compose a set of nine governing equations with ten independent variables, namely,
ρ_g – the density in the gaseous phase,
P – the pressure in the gaseous phase,
T – the temperature in the gaseous phase,
U_g – the velocity of the particles in the gaseous phase
x – the displacement in the gaseous phase,
ρ_c – the density in the cellular material,
σ_x – the normal stress (in the x-direction) in the cellular material,
U_c – the velocity of the particles in the cellular material,
ε_x – the strain (in the x-direction) in the cellular material and,
S – the displacement in the cellular material.

Consequently, in order to have a closed set of equations, which, in principle, could be solved, there is a need for an additional equation. The additional equation is the constitutive relation which is known more commonly as the stress-strain relation, i.e.,

$$\varepsilon_x = \varepsilon_x(\sigma_x). \tag{10}$$

It is this equation which dictates how materials, including cellular materials, will respond to a sudden load imposed by a head-on colliding shock wave. Some aspects of its importance are discussed in the next section.

The above listed set of ten governing equations must be solved simultaneously for the following boundary conditions,

$$U_g(h_g = H_g, t) = U_c(h_c = 0, t), \tag{11a}$$

$$U_c(h_c = H_c, t) = 0. \tag{11b}$$

The first condition (11a) implies that the velocities of the gaseous phase and the cellular material at the interface between them are identical, and the second condition (11b) implies that the rear face of the cellular material which is supported by the shock tube end-wall, see Fig. 1, has a zero velocity at all times.

The value of $U_c(h_c = 0, t)$ which appears in the right hand side of (11a) can be evaluated by the following equation,

$$\frac{\partial U_c(h_c = 0, t)}{\partial t} = \frac{2A}{\Delta h_g + \Delta h_c}(P_g(h_g = H_g, t) - \sigma_x(h_c = 0, t)). \tag{11c}$$

5. The stress-strain relation

Cellular materials, unlike solids, cover a very wide range of physical properties. For example (see Gibson and Ashby 1988), while the density of solid materials ranges from about 900 to about 20000 kgm/m^3, relative densities of cellular materials, such as ultra-low-density foams, can be as low as 0.001. Similarly, while Young's modulus in true solids ranges from about 10^3 to about 10^6 MN/m^3 that of special elastomer foams can be as low as 10^{-3} MN/m^3 while that of typical polymer foams is about 1 MN/m^3. These facts categorize some of the cellular materials (especially foams) as physically and geometrically non-linear.

Physical non-linearity means that due to the physical properties of the material, the relation between the stress and the strain is not linear, i.e., Hook's law does not apply to them. Geometrical non-linearity means that the material can experience large deformations and as a consequence instead of the linear relations between the strain and the deformation more complex relations must be used. The fact that cellular material are physically and geomerically non-linear specializes their stress-strain relations in comparison with true solids. For example, while the stress-strain relation of a solid polyurethane (in its elastic range) is:

$$\sigma_x = E_s\varepsilon_x$$

a cellular material made of the same material results in a situation in which the stress-strain relation (in the elastic range) consists of three domains: namely, the linear elastic; the elastic buckling; and the densification domains. In addition, the polyurethane foam behaves as an elastic non-linear material, which can experience large deformations.

Similarly, while the stress-strain relation of a solid rubber, which is an elastic non-linear material is

$$\sigma_x = \frac{E_s}{2(1+\nu_s)}\left[1 - \varepsilon_x - \frac{1}{(1-\varepsilon_x)^2}\right]$$

where ν_s is the Poisson ratio, and it is limited to deformations up to 60 %, in cellular material made of rubber (i.e., foamed rubber) the stress-strain relation again consists of the above mentioned three domains, and only in the third domain, i.e., the densification domain, does the above stress-strain relation hold. In addition, foam rubbers can experience deformations significantly larger than 60 %.

Thus, in summary when cellular materials are made of true solids the physical properties (density, Young's modulus, etc.) are significantly changed. This in turn results in, especially in polymers, a drastic change in the stress-strain relations and their capability to experience large deformations.

As will be shown subsequently, the stress-strain relation of a cellular material determines whether or not a compression wave, propagating inside it, can transform to form a shock wave.

6. Transformation of a compression wave to a shock wave

When the incident shock wave collides head-on with the front edge of a cellular material a compression wave is transmitted into the cellular material, as shown in Fig. 1b. Nowinski (1965) showed that each pulse in the compression wave, namely a Riemann wave, propagates at a velocity, c, given by,

$$c = U_c + \lambda_x c_0$$

where c_0, the shift rate of the disturbance, is given by (see Courant and Friedrichs 1948)

$$c_0 = \sqrt{\frac{1}{\rho_c^0}\frac{\partial \sigma_x}{\partial \lambda_x}}. \tag{12}$$

It should be noted here that c_0 is not the local speed of sound in the cellular material. Courant and Friedrichs (1948) showed that based on the following definition of the local speed of sound,

$$a^2 = -\frac{\partial \sigma_x}{\partial \rho_c}$$

the acoustic impedance, z, can be expressed as,

$$z = \rho_c^0 c_0 = \rho_c a$$

thus, the local speed of sound is

$$a = c_0 \frac{\rho_c^0}{\rho_c}.$$

Hence, the local speed of sound, a, is identical to the shift rate of the disturbance, c_0, only in incompressible materials.

It is evident from (12) that the stress-strain relation of the cellular material is the key parameter in determining whether or not pressure pulses, generated at early time of the compression of cellular materials, propagate slower or faster than those generated at later times. When pulses generated at early time of the compression process cellular materials propagate at a smaller velocity than those produced at later times, the formation of a shock wave in the cellular material is imminent. Nowinski (1965) showed that in order for the compression waves to transform to a shock wave, the stress-strain relation of the cellular material must fulfill the following two requirements.

$$\text{(1)} \quad \frac{\partial \sigma_x}{\partial \lambda_x} > 0, \tag{13a}$$

$$\text{(2)} \quad \frac{\partial^2 \sigma_x}{\partial \lambda_x^2} > 0. \tag{13b}$$

It is important to note here that the transformation of the compression waves to a shock wave, when the above two conditions are met by the stress-strain relation of the investigated cellular material, does not occur instantaneosly at $X = 0$. It occurs at some distance downstream of the leading edge of the cellular material. The distance, measured from the leading edge of the cellular material,to the location where the compression waves transform to a shock wave, S_s, could be calculated for a given $\sigma_x = \sigma_x(\varepsilon_x)$ relation, using the procedure outlined by Mazor (1989).

Hence, in the problem under consideration, shown schematically in Fig. 1, the compression waves can transform to a shock wave, provided the above two conditions are met, if in addition,

$$S_s < L_0$$

where S_s is the distance required for the compression waves to transform to a shock wave and L_0 is the initial length of the cellular material.

As will be shown in the subsequent parts of this study, there is a great uncertainty regarding the nature of the waves which are developed inside the cellular material even though the above mentioned three conditions, which theoretically are sufficient for the transformation of the compression waves to a shock wave, are fulfilled. The reason for this is yet to be understood and explained.

7. The pressure on the shock tube end-wall

Cellular materials are increasingly used structurally as cushioning and in systems for absorbing the kinetic energy from impacts. Consequently, it is of interest to check whether the arrangement shown in Fig. 1a could be used in order to reduce the shock wave load on the end-wall of the shock tube.

The pressure acting on the end-wall of the shock tube is that obtained by the transmitted wave, compression or shock, which collides head-on with the end-wall and reflects from it backwards. This is shown schematically in Figs. 3a and 3b where for simplicity, the transmitted compression waves have transformed to a shock wave, S_t, which upon its head-on collision with the end-wall, reflects backwards as a reflected shock wave, S_{tr}.

The transmitted shock wave, S_t, propagates towards state (I) of the cellular material and induces behind it, a new state, state (II). The reflected shock wave, S_{tr}, propagates towards state (II) and changes into state (III). Hence, the actual pressure acting on the end-wall of the shock tube, P_w, is the stress developed in state (III), i.e.,

$$P_w = \sigma_{x_{III}}.$$

Using simple gas dynamics and solid mechanics considerations it can be shown that

$$P_w = P_{g_5} + \frac{\rho_{c_{II}} - \rho_{c_I}}{\rho_{c_{III}} - \rho_{c_I}} \rho_{c_{III}} V_{c_t} U_{c_{II}} \tag{14}$$

where P_w is the pressure acting on the end-wall of the shock tube, P_{g_5} is the gas pressure acting on the front edge of the cellular material, ρ_{c_I}, $\rho_{c_{II}}$, and $\rho_{c_{III}}$ are the densities of the cellular material in states (I), (II), and (III), respectively, V_{c_t} is the velocity of the head of the transmitted compression wave and $U_{c_{II}}$ is the velocity of the cellular material in state (II).

Equation (14) indicates that the dominant parameters in determining the magnitude of P_w are the compressibility of the cellular material $\frac{\rho_{c_{II}} - \rho_{c_I}}{\rho_{c_{III}} - \rho_{c_I}}$ and its stiffness E_c since it determines both the velocity of the head of the transmitted compression wave, V_{c_t}, and the velocity of the front edge of the foam $U_{c_{II}}$, through the following relations:

$$V_{c_t} = \sqrt{\frac{E_c}{\rho_{c_I}}} \tag{15}$$

and

$$U_{c_{II}} = \frac{P_{g_5} L_0}{E_c \Delta t}. \tag{16}$$

Equation (16) is based on the experimental observation that $U_{c_{II}}$ is constant, and hence

$$\sigma_{x_{II}} = P_{g_5}. \tag{17}$$

Inserting (15) and (16) into (14) results in:

$$P_w = P_{g_5}\left[1 + \sqrt{\frac{\rho_{c_{III}}^2}{\rho_{c_I}}\frac{\rho_{c_{II}} - \rho_{c_I}}{\rho_{c_{III}} - \rho_{c_I}}\frac{L_0}{E_c^{\frac{1}{2}}\Delta t}}\right]. \tag{18}$$

Two extreme cases which result in the upper and the lower bounds on the magnitude of P_w can be deduced from the above expressions, they are:

(1) $E_c \rightarrow \infty$, i.e., an absolutely rigid cellular material; for this case (18) results in $P_w \rightarrow P_{5_g}$; in addition, (16) yields $U_{c_{II}} \rightarrow 0$; this in turn implies that $P_{5_g} \rightarrow P_{5_s}$ where P_{5_s} is the pressure which would have been obtained behind the reflected shock wave had it collided head-on with the end-walls of the shock tube without the presence of a cellular material; thus for this case we obtain $P_w \rightarrow P_{5_s}$.
(2) $E_c \rightarrow 0$, i.e., an absolutely flexible cellular material; for this case (18) results in $P_w \rightarrow \infty$.

Since in reality, E_c of cellular material is finite, i.e., $0 < E_c < \infty$ one must conclude that

$$P_w > P_{5_s}. \tag{19}$$

Hence, the pressure load exerted on the end-wall of a shock tube as a result of a head-on colliding shock wave, cannot be reduced by means of a cellular material. On the contrary, since the modulus of elasticity of cellular materials, E_c, is relatively low, the presence of cellular materials near the end-wall of the shock tube acts as pressure amplifiers.

This fact, which has been verified experimentally by many investigators, e.g., Gelfand et al. (1983), Gvodeva et al. (1985), Korobeinikov and Utriew (1989), and Skews et al. (1992), could be interpreted in the following way. Without a cellular material the pressure developing at the end-wall of the shock tube is the stagnation pressure of the shock induced flow in state (2). However, when a cellular material is mounted in the shock tube, the pressure developing at the end-wall is caused, in addition, from the stagnation of the accelerated cellular material there. As can be seen from (3) the smaller is E_c the higher is the shock induced velocity in the cellular material, $U_{c_{II}}$. As a consequence, the pressure developed while stagnating the cellular material is higher.

8. The numerical scheme

In subsequent parts of this study the shock wave interaction with specific cellular materials will be investigated numerically and experimentally. Since the numerical simulations of all the subsequent parts of this study are based on the same scheme, and in order not to repeat it in every part it is outlined in the following.

Equations (1) to (10) form a set of partial differential equations that cannot be solved analytically. Therefore, one must resort to numerical solutions. Over the past few decades, a variety of numerical schemes, capable of handling flow discontinuities have been developed. During the course of the present research the artificial viscosity method, developed originally by von Neumann and Richtmeyer (1950) was adopted. Written in Lagrangian coordinates, the resulting numerical scheme is very convenient in handling moving interfaces such as that between the gaseous phase and the cellular material. In addition, a wide range of constitutive relations of cellular materials can be treated with the same scheme and minor changes only.

When introducing the artificial viscosity terms into the governing equations only equations in which pressures and stresses appear. Namely, the conservation equations of linear momentum and the equation of state are changed. Consequently, only the modified version of these equations both for the gaseous phase and the cellular material are given.

By adding the artificial viscosity term to the conservation equation of linear momentum (3) and the equation of state (5) of the gaseous phase they respectively become,

$$\frac{\partial U_g(h_g, t)}{\partial t} = -A\frac{\partial}{\partial h_g}(P(h_g, t) + q_g(h_g, t)) \tag{20}$$

and

$$R\frac{\partial T(h_g, t)}{\partial t} = -(P(h_g, t) + q_g(h_g, t))\frac{\partial V_g(h_g, t)}{\partial t} \tag{21}$$

where the artificial viscous pressure term for the gas is,

$$q_g = \begin{cases} \dfrac{(C_g \Delta h_g)^2}{V_g(h_g, t)}\left(\dfrac{\partial V_g(h_g, t)}{\partial t}\right)^2, & \dfrac{\partial V_g(h_g, t)}{\partial t} < 0 \\ 0, & \dfrac{\partial V_g(h_g, t)}{\partial t} \geq 0 \end{cases} \tag{22}$$

where C_g is a nondimensional constant which controls the shock wave numerical thickness in the gaseous phase. The commonly used values for C_g range between 1.5 and 2, thus extending the shocks over 3–5 intervals of Δh_g. It can be seen from (22) that the artificial viscous pressure term q_g becomes significant only in regions where shock waves are present.

Similarly, if an artificial viscosity term is added to the conservation of linear momentum equation (9b) of the cellular material, it becomes,

$$\frac{\partial U_c(h_c, t)}{\partial t} = -A\frac{\partial}{\partial h_c}(|\sigma_x(h_c, t)| + q_c(h_c, t)) \tag{23}$$

where the artificial viscous pressure term for the cellular material is:

$$q_c = \begin{cases} \dfrac{(C_c \Delta h_c)^2}{\lambda_x(h_c, t)}\left(\dfrac{\partial \lambda_x(h_c, t)}{\partial t}\right)^2, & \dfrac{\partial \lambda_x(h_c, t)}{\partial t} < 0 \\ 0, & \dfrac{\partial \lambda_x(h_c, t)}{\partial t} \geq 0 \end{cases} \tag{24}$$

where C_c is a nondimensional constant which, similar to C_g in (22), controls the shock wave numerical thickness in the cellular material. Similar to C_g, values of C_c ranging between 1.5 and 2 ensure that the shock waves extend over 3–5 intervals of Δh_c. Equation (24) implies that the artificial viscous stress term q_c becomes significant in the flow regions where shock waves are present.

Equations (1), (2), (20), (4), (21), (6), (7), (8), (23), and (10) were transformed into a set of central finite difference equations. As a result, a solution of second-order accuracy in space and time (i.e., $O(\Delta t)^2 + O(\Delta h)^2$) was obtained. Thus, knowing all the fluid properties up to (and including) some time $t = n\Delta t$ where n is an integer, it was possible to calculate the fluid properties at the next time level, i.e., $t = (n + 1)\Delta t$. For the spatial differencing, a staggered grid was used. The velocity and the Eulerian distance were computed at $J\Delta h$, while the rest of the fluid properties, such as pressure, density, etc. were calculated at $(J + 1/2)\Delta h$, where $J = 1, 2, \ldots, N$ and N is the total number of space intervals. Details regarding the numerical scheme can be found in Mazor (1989).

9. Summary and conclusions

The equations describing the head-on collision of planar shock waves with cellular materials which fill the end of a shock tube were developed.

A simple gasdynamic and solid mechanics investigation of the developed governing equations revealed that cellular materials cannot be used to reduce the pressure loadings imposed by shock waves on the end-wall of a shock tube. On the contrary, cellular materials act as pressure amplifiers.

In addition, the conditions for the compression wave (which is transmitted into the cellular material) to transform to a shock wave, inside the cellular materialwere introduced.

In subsequent papers, the interaction of planar shock waves with specific cellular materials, e.g., elastomeric open cell foams, elastomeric closed cell foams, honeycombs etc., will be investigate both numerically and experimentally.

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